

# HIGGS-SECTOR SOLITONS

**C. Bachas** \*

*Centre de Physique Théorique  
Ecole Polytechnique  
91128 Palaiseau, FRANCE*

and

**T.N. Tomaras** †

*Physics Department, University of Crete  
and Research Center of Crete  
714 09 Heraklion, GREECE*

## ABSTRACT

We establish the existence of static, classically-stable, winding solitons in renormalizable three-dimensional gauge models, with topologically trivial target space and vacuum manifold. They are prototypes for possible analogous particle-like excitations in the higgs sector of the standard electroweak theory.

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\*e-mail address: bachas@orphee.polytechnique.fr

†e-mail address: tomaras@plato.iesl.forth.gr

Solitons are perhaps the most spectacular non-perturbative feature of field theories [1]. Their presence is often guaranteed by a conserved topological number to which one can associate some minimum mass or energy. Although the standard electroweak theory has no such topologically-stable solitons, it could still possess classically-stable excitations that are sufficiently long-lived to be relevant for cosmology. The recent discussions of electroweak vortex strings [2] [3] underscore indeed our ignorance of the non-topological excitations that can arise in gauge models. In this letter we present some evidence for the existence of non-topological <sup>‡</sup> particle-like solitons in an extended higgs sector.

Most earlier discussions of electroweak solitons [7] [8] [9] [10] assumed a strongly-interacting higgs sector, whose would-be goldstone bosons are described at low energy by an effective non-linear  $\sigma$ -model. It has been argued that the corresponding non-renormalizable lagrangian can have stable solutions, which are identified with the technibaryons of an underlying technicolor model, and whose distinguishing characteristic is the non-trivial wrapping of the target  $SU(2)$  manifold [11]. This, of course, is at best a phenomenological description, since the properties of the soliton cannot be calculated reliably in a semiclassical expansion. The question that arises naturally is whether such winding excitations can be classically stable even for a *weakly-coupled* higgs sector. In order to guide our thinking let us distinguish three sources of potential instability:

- the winding can be undone if the higgs field passes through zero,
- the evolution of the gauge fields can take us to a winding-vacuum state plus radiation [12] [8] , and
- scalar-field excitations lose their energy by shrinking to zero size [13].

The first (higgs) instability, which is not considered in the non-linear  $\sigma$ -model limit, imposes a lower bound on the physical-scalar mass times the would-be soliton size:

$$m_H \rho > C . \quad (1)$$

This follows by requiring the loss in potential energy  $\sim \lambda v^4 \rho^3$  to exceed the gain in gradient energy  $\sim v^2 \rho$  when trying to make the higgs vanish in the interior of the soliton. The second (gauge) instability puts on the other hand an upper bound on the gauge-boson mass times the would-be soliton size [9] [10]:

$$m_W \rho < C' . \quad (2)$$

This follows by requiring the loss in weak-magnetic energy  $\sim 1/g^2 \rho$  to exceed the gain in gradient energy when trying to turn on continuously weak gauge fields to reach the winding-vacuum state. Here  $v$  is the vacuum expectation value of the higgs,  $\lambda$  its quartic self-coupling,  $g$  the gauge coupling, and  $C, C'$  numerical constants. Taken together the above two bounds make a priori unlikely the existence of winding solitons in the perturbative minimal standard model. The prospects, however, are better in the presence of an extra higgs since the gauge instability, and consequently the bound (2), are in this case absent. The validity of the above arguments was illustrated explicitly in a toy two-dimensional model [6].

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<sup>‡</sup>Unlike Q-balls [4] the non-topological solitons discussed here are static and uncharged [5] and owe their stability to the dynamical exclusion of some region of configuration space [6].

The issue which did not arise in this toy model is what can cure the third (scale) instability and fix the soliton size. Barring quantum effects [14] [15] [7], which would take us outside the semiclassical treatment, a stabilizing role can only be played by the electro-weak magnetic fields which are induced by the winding currents. There is an encouraging though not conclusive argument that this may indeed happen in a two-higgs standard model. It is a paraphrasing of previous suggestions to stabilize the scale of the soliton with extra heavy (hidden) gauge bosons [16]. To simplify the argument let us freeze the magnitudes of the two doublets to some common value  $v$ , and let  $U$  be the relative  $SU(2)$  phase. We can obtain an energy functional for  $U$  by solving the classical gauge-field equations in a  $\partial/m_W$  expansion. The first two terms of this energy functional are precisely those of the Skyrme model which has indeed stable winding excitations [11]. Unfortunately the argument is inconclusive because the size of the would-be soliton turns out to be  $\rho \sim 1/m_W$ , thus invalidating our derivative expansion. The issue must thus be decided numerically in four dimensions [17]. The purpose of this letter, on the other hand, is to demonstrate analytically that gauge fields do stabilize winding solitons in an analogous three-dimensional model.

Our starting point is the three-dimensional  $O(3)$  non-linear  $\sigma$ -model

$$S_0 = \frac{v^2}{2} \int d^3x \partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n} \quad (3)$$

where  $\mathbf{n}$  is a three-component scalar field subject to the constraint

$$\mathbf{n} \cdot \mathbf{n} = 1. \quad (4)$$

We can solve the constraint by a stereographic projection of the three sphere onto the complex plane:

$$n_1 + in_2 = \frac{2\Omega}{1 + |\Omega|^2} \quad ; \quad n_3 = \frac{1 - |\Omega|^2}{1 + |\Omega|^2}. \quad (5)$$

It is well known that the above model has static winding soliton solutions [18] given by holomorphic functions  $\Omega(z)$  where  $z = x_1 + ix_2$ . The solitons are classified by the number of times two-space wraps around the target sphere:

$$N = \frac{1}{\pi} \int d^2x \frac{\bar{\partial}\bar{\Omega}\partial\Omega - \bar{\partial}\Omega\partial\bar{\Omega}}{(1 + |\Omega|^2)^2}, \quad (6)$$

where  $\partial$  here stands for  $\frac{\partial}{\partial z}$ . The simplest solution,

$$\Omega^{sol} = \frac{\rho e^{i\theta}}{z - z_0} + w_0, \quad (7)$$

describes a soliton with unit topological charge and energy  $E^{sol} = 4\pi v^2$ . It is characterized by six real parameters reflecting the invariance of the underlying equations under the two-dimensional conformal group  $SL(2, \mathbb{C})$ . The complex parameter  $w_0$  is in fact fixed by the choice of boundary conditions at infinity:  $w_0 = 0$  if  $\mathbf{n} \rightarrow (0, 0, 1)$ . The remaining four collective coordinates correspond to translations,  $U(1)$  rotations, and scale transformations of the soliton.

Let us next relax the non-linear constraint (4) by introducing a mexican-hat potential. By Derrick's scaling argument [13] winding configurations are now unstable against

shrinking to zero size. Since we are interested in *renormalizable* models, we are not allowed to stabilize the size of the soliton with explicit higher-derivative terms in the action [19]. We must thus try to evade Derrick's argument by introducing gauge interactions. The simplest possibility is to gauge a  $U(1)$  subgroup of the global  $O(3)$  symmetry of the model. The corresponding gauge field can furthermore be massive without violating renormalizability, provided it couples to a conserved current. We are thus led to consider the following action:

$$S = \int d^3x \left[ \frac{1}{2} |(\partial_\mu + ieA_\mu)(\Phi_1 + i\Phi_2)|^2 + \frac{1}{2} \partial^\mu \Phi_3 \partial_\mu \Phi_3 - V(\Phi) - \frac{1}{4} \mathcal{F}_{\mu\nu}^2 + \frac{m^2}{2} A^\mu A_\mu \right] , \quad (8)$$

with

$$V(\Phi) = \frac{\lambda}{4} (\sum_a \Phi_a \Phi_a - v^2)^2 + \frac{\kappa^2}{8} (\Phi_3 - v)^4 . \quad (9)$$

Our choice of scalar potential is not the most general one consistent with the symmetry of the model, but was dictated by later convenience. Likewise, the mass of the gauge field could arise from its coupling with an extra complex scalar, but such a complication will not be necessary for our purpose. The model defined by eqs. (8) and (9) has trivial topology, both in its scalar manifold and in its vacuum sector. It reduces however to the ungauged  $O(3)$  non-linear  $\sigma$ -model in the naive

$$\lambda \rightarrow \infty \quad \text{and} \quad e, \kappa \rightarrow 0 \quad (10)$$

limit. Our strategy will therefore be to show that for some range of parameters it has classically-stable solitons, which are small deformations of the configuration (7) with  $w_0 = 0$  and fixed size.

To this end, let us decompose the scalar triplet field into a radial and an angular part:  $\Phi_a = F n_a$ , with  $\mathbf{n}$  a vector of unit length which can be expressed through  $\Omega$  as in eq. (5). Working in units of the gauge-boson mass,  $m = 1$ , and rescaling:  $F \rightarrow F/\sqrt{2\lambda}$  and  $A_\mu \rightarrow A_\mu/\sqrt{2\lambda}$ , brings the action to the form

$$S = \frac{1}{2\lambda} \int d^3x \left[ \frac{1}{2} (\partial_\mu F)^2 + \frac{1}{2} F^2 |(\partial_\mu + i\tilde{e}A_\mu)(n_1 + in_2)|^2 + \frac{1}{2} F^2 (\partial_\mu n_3)^2 - \frac{1}{4} (F^2 - m_H^2)^2 - \frac{\tilde{\kappa}^2}{8} (Fn_3 - m_H)^4 - \frac{1}{4} \mathcal{F}_{\mu\nu}^2 + \frac{1}{2} A^\mu A_\mu \right] , \quad (11)$$

with

$$m_H \equiv \sqrt{2\lambda}v , \quad \tilde{e} \equiv e/\sqrt{2\lambda} \quad \text{and} \quad \tilde{\kappa} \equiv \kappa/\sqrt{2\lambda} . \quad (12)$$

The above rewriting demonstrates that  $\tilde{\kappa}, \tilde{e}$  and  $m_H$  are the only classically-relevant parameters of the model. The quartic scalar coupling  $\lambda$  on the other hand plays the role of Planck's constant  $\hbar$ , and can be taken to zero independently in order to approach a semi-classical limit. The existence of classically-stable winding solitons will not therefore be tied to the presence of a strongly-interacting scalar sector.

To look for static minima of the energy we will proceed in two steps: we first keep the angular degree of freedom  $\mathbf{n}$  fixed and time-independent, and minimize the energy

with respect to the radial and gauge fields  $F$  and  $A_\mu$ . Assuming these stay close to their vacuum values one finds:

$$F \simeq m_H \left[ 1 - \frac{1}{2m_H^2} \partial_i \mathbf{n} \cdot \partial_i \mathbf{n} \right] , \quad (13)$$

$$A_0 = 0 , \quad \text{and} \quad A_k(x) \simeq 2\tilde{e}m_H^2 \int d^2y G_{kl}(x-y) J_l(y) , \quad (14)$$

where

$$J_l = \frac{1}{2} (n_2 \partial_l n_1 - n_1 \partial_l n_2) \quad (15)$$

is the U(1) current of the scalars, and

$$G_{kl}(x) = \int \frac{d^2p}{(2\pi)^2} e^{-i\vec{p}\vec{x}} \frac{\delta_{kl} + p_k p_l}{\vec{p}^2 + 1} \quad (16)$$

is the two-dimensional massive Green function. Consistency of our approximation requires that

$$\frac{1}{m_H \rho} \ll 1 , \quad \tilde{\kappa} m_H \rho \ll 1 , \quad \text{and} \quad \tilde{e} m_H \min(\rho, 1) \ll 1 , \quad (17)$$

with  $\rho$  the typical scale over which  $\mathbf{n}$  varies. These conditions ensure in particular that  $F - m_H \ll m_H$ , and that  $\tilde{e} A_i \mathbf{n} \ll \partial_i \mathbf{n}$ . They give a precise meaning to the naive limit, eq. (10). Since  $\rho$  will be determined dynamically, we must a posteriori check that these constraints can indeed be satisfied.

Eliminating  $F$  and  $A_\mu$  with the help of eqs. (13) and (14) we arrive at an energy functional that depends only on the angular degrees of freedom. It is of the form

$$E = E_0 - \mathcal{E} \quad (18)$$

where

$$E_0 = \frac{m_H^2}{2\lambda} \int d^2x \frac{1}{2} \partial_i \mathbf{n} \cdot \partial_i \mathbf{n} \quad (19)$$

is the energy in the non-linear  $\sigma$ -model limit, while

$$\begin{aligned} \mathcal{E} = \frac{m_H^2}{2\lambda} \left[ \frac{1}{m_H^2} \int d^2x \left( \frac{1}{2} \partial_i \mathbf{n} \cdot \partial_i \mathbf{n} \right)^2 - \frac{1}{8} \tilde{\kappa}^2 m_H^2 \int d^2x (n_3 - 1)^4 \right. \\ \left. + \tilde{e}^2 m_H^2 \int d^2x \int d^2y J_i(x) G_{ik}(x-y) J_k(y) \right] , \end{aligned} \quad (20)$$

is a small perturbation under the above assumptions.

Let us here pause for a minute and consider a simple calculus problem: we are asked to minimize a function of two variables  $G(u, v) = G_0(u, v) - \mathcal{G}(u, v)$ , where  $G_0$  has a line of degenerate minima along the  $u$  axis, while  $\mathcal{G}$  is a small perturbation. Minimizing first with respect to  $v$  yields a line  $\bar{v}(u)$  which lies a priori close to the  $u$ -axis. Along this line one finds easily

$$G(u, \bar{v}(u)) \simeq \mathcal{G} - \frac{1}{2} \mathcal{G}' (G_0'')^{-1} \mathcal{G}' + o(\mathcal{G}^3) , \quad (21)$$

where the primes stand for derivatives with respect to  $v$  and all the functions on the right-hand side are evaluated at  $v = 0$ . As shown by this formula, for the expansion in powers of  $\mathcal{G}$  to be valid  $G_0''$  must stay bounded away from zero, meaning that the valley must not become too shallow in the transverse direction. In this case the first term of the series dominates, and the minima of the function  $G$  are given by the minima of the perturbation  $\mathcal{G}$  along the  $u$  axis.

Going back to the energy functional, eq.(18), one notes that the role of  $u$  is played by the zero modes of  $\Omega^{cl}$ , which is a local minimum of  $E_0$ , while the role of  $v$  is played by the infinite number of transverse fluctuations. Let us write  $\mathbf{n} = \mathbf{n}^{cl} \sqrt{1 - (\delta\mathbf{n})^2} + \delta\mathbf{n}$  with  $\mathbf{n}^{cl} \cdot \delta\mathbf{n} = 0$ , and consider fluctuations which can be normalized on a sphere of radius  $\rho$ , i.e. with respect to the inner product

$$\langle \delta\mathbf{n}, \delta\mathbf{n}' \rangle \equiv \int d\mu(x) \delta\mathbf{n} \cdot \delta\mathbf{n}' , \quad \text{with} \quad d\mu(x) \equiv \frac{1}{\pi\rho^2} \frac{d^2x}{(1 + |x|^2/\rho^2)^2} . \quad (22)$$

In the vicinity of  $\mathbf{n}^{cl}$  the energy reads in an obvious notation:

$$E - E^{cl} \simeq -\mathcal{E}(\mathbf{n}^{cl}) - \int d\mu \frac{1}{2} \delta\mathbf{n}^T \cdot E_0'' \cdot \delta\mathbf{n} - \int d\mu \mathcal{E}' \cdot \delta\mathbf{n} + o(\delta\mathbf{n}^3, \mathcal{E}\delta\mathbf{n}^2) . \quad (23)$$

The matrix of quadratic fluctuations  $E_0''$ , has been shown in ref. [20] to have a discrete spectrum:  $\lambda^{(j,\alpha)} = j(j+1) - 2$ , where  $j = 1, 2, \dots$  and  $\alpha$  labels some finite degeneracy. It is furthermore straightforward to check that with the inner product (22) the first variation of the perturbation,  $\mathcal{E}'$ , can be normalized. The analysis of the calculus problem is under these conditions easily extended to show that we need only minimize the energy in the space of zero modes of the unperturbed soliton, since transverse fluctuations affect the equations at higher orders.

Translation and rotation invariance ensures in fact that the energy does not depend on the  $U(1)$ -orientation and position. For any non-zero value of  $\tilde{\kappa}$  on the other hand, the energy is infinite unless  $w_0 = 0$ . The only relevant collective coordinate is thus the scale, and after a straightforward calculation we find

$$E(\rho) = \frac{2\pi m_H^2}{\lambda} \left[ 1 + \frac{1}{6} \tilde{\kappa}^2 m_H^2 \rho^2 - \frac{4}{3m_H^2 \rho^2} - \tilde{e}^2 m_H^2 \rho^2 \int_0^\infty dx \frac{x^3 K_0^2(x)}{x^2 + \rho^2} \right] , \quad (24)$$

with  $K_0$  the modified bessel function. The shape of the function  $E(\rho)$ , up to overall multiplicative and additive factors, depends only on the two parameters

$$a \equiv \frac{\tilde{\kappa}^2}{\tilde{e}^2} \quad \text{and} \quad b \equiv \frac{1}{\tilde{e}^2 m_H^4} . \quad (25)$$

In the region above the thick line of fig.1  $E$  grows monotonically with  $\rho$  so that, to the extent that our approximations are valid, we conclude that the would-be soliton is

unstable against shrinking. In the region below this thick line, on the other hand, the function develops a local minimum at some size  $\bar{\rho}(a, b)$  at which the soliton is stabilized. The tangents to the boundary of stability are lines of constant  $\bar{\rho}$  as shown in the figure. To complete our proof of the existence of stable solitons, we must still make sure that conditions (17) can be satisfied. This can however always be arranged by taking  $m_H$  sufficiently large, while keeping  $a$  and  $b$  fixed at any point below the thick line. Determining the complete region of stability in the  $(m_H, \tilde{\kappa}, \tilde{e})$  space requires a numerical investigation, which is beyond the scope of the present letter.

Let us conclude with a comment on the potential importance of such non-topological solitons, should they turn out to exist in the electroweak model. Since they would decay by quantum tunneling, they could be stable on cosmological time scales. Furthermore their expected size is  $\sim 1/m_W$ , their expected mass in the  $TeV$  range, while their annihilation cross-section, being essentially geometrical, should be somewhat larger than weak cross-sections. These properties would make them serious candidates for cold dark matter in the universe.

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